

The Metric Space of the Earth information and Global GIS

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Abstract: One principal and basic issue of GIS is that what kind of metric space should be used. A metric space is a space defined by its measurement. We using different measurement under different conditions in GIS, for example, Euclidian distance in Euclidian space for small area while geodesic distance in Earth's ellipsoid space. With the development of GIS, the spatial mathematical foundation should no longer be confined within map projection and should find its way for global GIS and fit for spatial analysis in broad sense.

This article starts from the conception of metric space and demonstrate the metric space of earth spatial information. This article firstly discusses the conception of itself, secondly the conception of it in small area with low precision, then the conceptions under other special conditions. It also discusses some important foundational technical issues under certain metric space of earth spatial information. Integrated with important technical trend of Global-GIS, it discusses the significance of "sphere model" and the limitation of O-QTM partition, and then it points out the significance of "ellipsoid model" in global-GIS.

This article takes the creation of Voronoi diagram of the continents on the earth for example, and shows the importance of selecting proper spatial measurement. The authors have done much research on Map Algebra that is quite different from that of Tomlin. In this article, the authors will come up with experiments to show the correct and rigorous Voronoi diagram created with their Map Algebra, illustrating the conceptions and importance of spatial measurement and according metric space.

Based on above discuss, it comes up with the mathematical foundation in metric space, the theory and technology of visualization, the handling and analyzing of spatial data, etc. All these issues are a series of foundational work, which needs to be achieved and is cable of achievement. With the development of integration of global economy and concern of global environmental change, the geographical information science should pull more research on metric space and Global-GIS.

Keywords: metric space, Global-GIS, sphere model, ellipsoid model, Map Algebra

What kind of metric space should be adopted in GIS is a primary and basic issue of GIS. It is the basic and developmental work, which should gain a unitive recognition. With the integration of global economics and the concern on global environment change, GIS should do more research on the metric space of the earth information and GlobalGIS.

I.Metric Space

The metrication of size and shape of an object is the important base of spatial analysis, and it is the most important quantity feature of the space. The base of measurement is the metric.

1.The mathematical meaning of the metric[1]

In the reference[1],it gives the definition of metric. There different metric space with different metric definition and vice versa. So, even with the same point set of the same coordinates, they

have different spatial quantity feature in different metric space.

The effects **among** objects occur through the space, especially in the nearby space, so research on spatial neighborhood has become a hot point in GIS. The neighborhood is determined by the metric while the topological relation is not, although that the distance between them is zero is the prerequisite of topological correlation.

2. The Euclidian Space[1]

The essential feature of the Euclidian Space is the metric of space. Reference[1] also gives **the** definition of the Euclidian Space, it is the space which defines the following metric:

$$J(x, y) = \left[\sum_{i=1}^n (x_i - y_i)^2 \right]^{1/2} \quad (1)$$

The **geographical** space is often substituted with the Euclidian Space when the research area is small.

3. Distance between a point to a point set.

Definition 1: the distance $d(a, B)$ between a point to a point set is:

$$d(a, B) = \text{MIN}\{d(a, b)\} \quad \text{where } b \in B \quad (2)$$

B is named the reference. The function MIN is defined according to practical application.

4. Distance transformation

Definition 2: Distance transformation is a transformation(or procedure)that computes and marks the distance between the points to the reference.

When the object of distance transformation is the entities themselves and the reference is the non- entity point sets, this distance transformation is named internal distance transformation. Whereas, it is named external distance transformation. It is obvious that internal distance transformation is a comprehensive description of areal entities, and since the point and lines have no insides, the external distance transformation is used to describe the morphy.

The correct distance is obtained through according to the metric. If a approximate metric is adopted, the adaptability and the condition should be analyzed in detail in order to gain the validity and reliability. This has become both a academically rigorous and technically important issue.

II. The metric space of the Earth information

It is an important basic issue of how to measure the spatial information of the Earth. This is determined by the features of the space itself: the distribution, the movement and the interaction among them, which is almost what the conception of “geography space” contains. Added with the research area of the aerobiology, geology and others, the Earth space is made up of the multi- circle space. This description can only be understood but not technically applied. As to now, the metric space of earth information is the

mathematical definition of earth space.

1. The metric space of the Earth information.

The metric space of the Earth information can be named the spatial mathematical foundation of GIS, it contains following three aspects:

①The spatial reference of the earth information

The shape and size of the earth has been strictly defined by several geography subjects such as gravity study. The earth ellipsoid is the recognized spatial reference system by all kinds of subjects of geography, and supported by any of the international organizations. GIS should use the earth ellipsoid as the only reference system. Nowadays, parameters of the earth ellipsoid are constantly being precised even more forming the tendency of real time updating. It's important to the study of global space-time and space, but has not affected the geography entities which is developed along with science development. For a huge science system, the benchmark had better not be changed until it touches the critical line.

②The reference frame of the earth spatial information

The reference frame is usually determined by object and property of the problem. The study objects of geoscience always distribute as earth-circling layers. Two equal reference frame(Geocenter coordinate frame(X、Y、Z)and Geodesic coordinate frame (B、L、H)) are usually used in Geodesy, and sometimes plane-Angle or sphere coordinate are used to deal with some local problems. Considering parameters in Geodesic coordinate (B,L) are indeed 3-dimensional and objects of geoscience always locate around the surface layer of the earth, Geodesic coordinate is more convenient and stable than Geocenter coordinate.

③space measurement of earth information -- **geodesic distance in Earth's ellipsoid space**

Metric is the criterion of 量化. It lies on the request on properties, scope and precise of space and objects. Spatial analysis is an analysis in connection with space geometry quantity. The results of spatial analysis will be different while adopting different metric.

Measurement on ellipsoid space, according to differential geometry, should base on geodesic distance that is the shortest path on ellipsoid space. It's a 3D curve on ellipsoid surface. Plane of every point on the curve is vertical to ellipsoid surface and contain the normal of the point. Euclidean measurement cannot exactly modify GIS space. Even in local area, it is important to give precision limit for Euclidean measurement. Euclidean measurement is out of place for vast area. Measurement in GIS space should adopt the geodesic distance defined by Geodesic inverse problem (Besel) on ellipsoid surface. It's widely different from Euclidean measurement.

Global space in GIS should be the ellipsoid space and Geodesic coordinate is always used. Its metric space adopts geodesic measurement. Euclidean space described by Descartes plane plus z is only a low-precision approach to GIS space in local area, and can only be called special example. To make GIS space accord with Geoscience space meets great need of GIS development. Fang Yu, Zhou ChengHu, Jing GuiFei (reference[4],2001.6) and

Hu Peng, Hu YuJu(reference[5] 2001.3, reference[6] 2001.11) have point out this viewpoint for several times. It's clear in theory and must be a technique tendency. In some special environment, other metric definition should be used for example Manhattan distance in urban area and obstacle distance in obstacle space.

Metric space of earth information makes great effect on GIS. Metric space developing from Euclidean space to ellipsoid space will change the theory and technique of GIS greatly.

The basic theory of metric space being successfully settled, “no sew space database” could be just a virtual problem. Therefore 6 global scope problems hereinafter will have proper settlement.

- a. Global continues vision
- b. System , tolerance, unit of Multi-distinguish
- c. Multi-Dimension, source data global location frame
- d. Global precise measurement
- e. Correction of spatial analysis
- f. Construction of information system work

These are the ineluctable problems in large GIS.

1. Several important technique problems:

① **“Why not Geocentric coordinate?”**

The reference frame is usually determined by object and property of the problem. Geocentric coordinate is equivalent to Geodesic coordinate in principle and either can easily transform to the other. Both of them are in point, but objects in GIS always distribute around earth-circling surface. Geodesic coordinate frame that adopts earth ellipsoid surface as datum plane is more convenience for these data disposal while geocentric coordinate frame fits for planets, aerocrafts and other objects in extraterrestrial space. It' s obvious that the earth ellipsoid surface using B,L coordinate is a 2-dimension frame refer to the earth ellipsoid space and direction of normal is regarded as the 3rd dimension H. Sometimes assisting with 平面直角 or 球面坐标 frame can help to settle some local problems.

② **“Why suggest using distance-equal cylinder project for views^{5]} 、 [7]?”**

In some viewpoints, using distance-equal cylinder project should lead to the fault of low

precision. It's needless under this condition. There are 2 factors affecting precision, first area and angle distortion, second location precision. For maps, distortion will be bigger on high latitude, but for information system it will not use handwork to make measurement. In fact because of the coordinate frame lies on B,L, measurement on computer won't distort if the B,L coordinate is exact. The measurement is irrespective to which projection has been adopted, only relates to projection transform itself and precision of the source data. After source data has been selected, whether transform is exact will be the most important factor. The chiefly request is the consistency of every point between source map and destination and continuity in both-direction.

In spite of map or digital map, character point capture bases on the feature of the graph itself that will almost not be changed in the projection transform in the range of near area of a certain point. Therefore, 'leading to low precision' has no connection with view projection of distance-equal cylinder project.

Certainly, we should pay more attention to the image quality caused by projection on high lat.

③ aim of no sew technique

Physical no sew and vision no sew are the two forms of no sew technique. So called 'vision no sew' technology means seemingly no sew on screen. For example, there are two maps that have the border upon Gauss projection zone of 1:50000. The inner border of the map seems quadrangle (in fact curve). When moving, one of the maps could be rotated for a tiny dispatch angle between two meridians in computer memory, then there are no sew on the screen. However, because the dispatch angle between two meridians of the border upon Gauss projection zones is not a fixed value, a certain sew must appear on the borders of four border upon map, the same as M*N maps and high-dimension data. The 'vision no sew' technique is a static no sew for local area. It only adapt to single distinguish projection zone and its border zones. The real no sew technique need not only 'vision no sew' but also the continuity in all directions in topology meaning and precise matching on multi-分辨率. 6 global scope problems being settled can construct a physical no sew which is not just a skill.

IV. Main points of Ellipsoid Model of Global GIS scheme

This scheme bases on 'Global real-time Information metric space'. Any applied metric space will be adopted basing on the size of the area, precision request and feature of the objects; aiming at most application situation on earth surface to select a project scheme to the ellipsoid surface; meeting the need of most vision needs to select distance-equal cylinder project as the main project scheme; keeping the 1-1 relationship between earth space and it; implementing 2D and 3D vision basing on the project scheme; replacing line of vector model between two points in 2D Euclidean space with geodesic distance in ellipsoid space.

Points hereinafter should be noticed when entirely putting in practice:

1. Any applied metric space should be adopted basing on the size of the area, precision request and feature of the objects for data disposal, measurement and spatial analysis.

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- ① Ellipsoid model should be adopted in middle and large area and some dynamic

or high-precision condition.

This point has been accepted by many people. They change data in database to geography coordinate one after another. However many problems appear with the change. Difference between differ 分辨率 data, systematism damage, inconvenience caused by the data changing from familiar Euclidean space to a non-Euclidean space and so on, such problems do great harm to these changes.

② In small area and some static or low-precision condition, we can use traditional methods.

③ Aiming at some special objects, we can use some feasible metric space in local area.

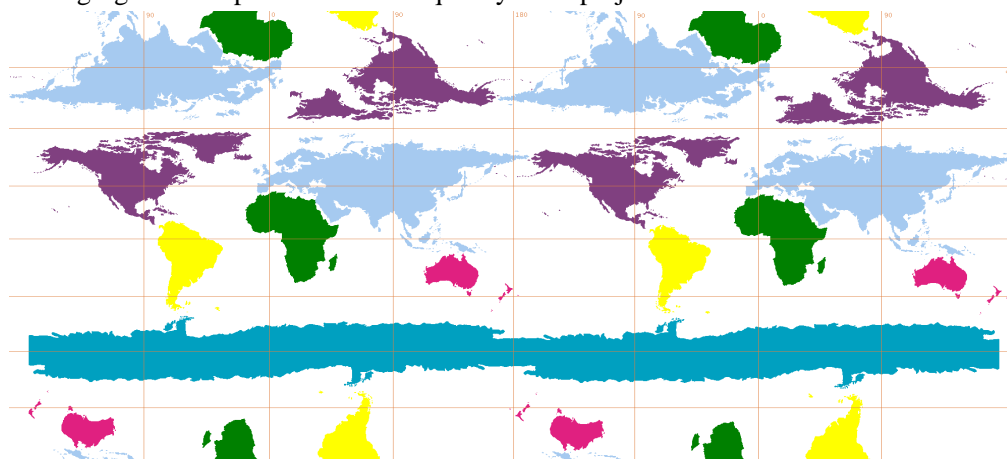
2. Meeting the need of most vision needs to select vision scheme

①meeting the need of most vision needs to select distance-equal cylinder project as the main project scheme, keeping the 1-1 relationship between earth space and it(defined in mathematics). Considering there is dinky difference between plane vision and curvature in large scale, and do no effect on man-machine interaction, measurement, analysis and other disposals, we can use it to put 2D and 3D vision to practice.

② Suggest using improved distance-equal cylinder project

A sphere(ellipsoid surface) got rid of a point is equal to a plane in topology. That is, a plane map is 1-1 corresponding and both-direction continues to the sphere except the certain point, so we can use two or three maps to express the ellipsoid surface and at the same time keep topology feature. Accordingly, every point on ellipsoid and its near zone can find the corresponding area on the map combination above.

Referencing figure 2 : improved distance-equal cylinder project



The recommendation map projection for global message.

In fact, it is just a ordinary distance-equal cylinder project(B:S90° --N90° ,L:0° --360°) extends from the two poles to equator. The only fault of it is that the points of poles change to lines.

For Global GIS, advantage is not only vision, but also settling aforementioned 6 Global problems.

3. Expression and organization of spatial data

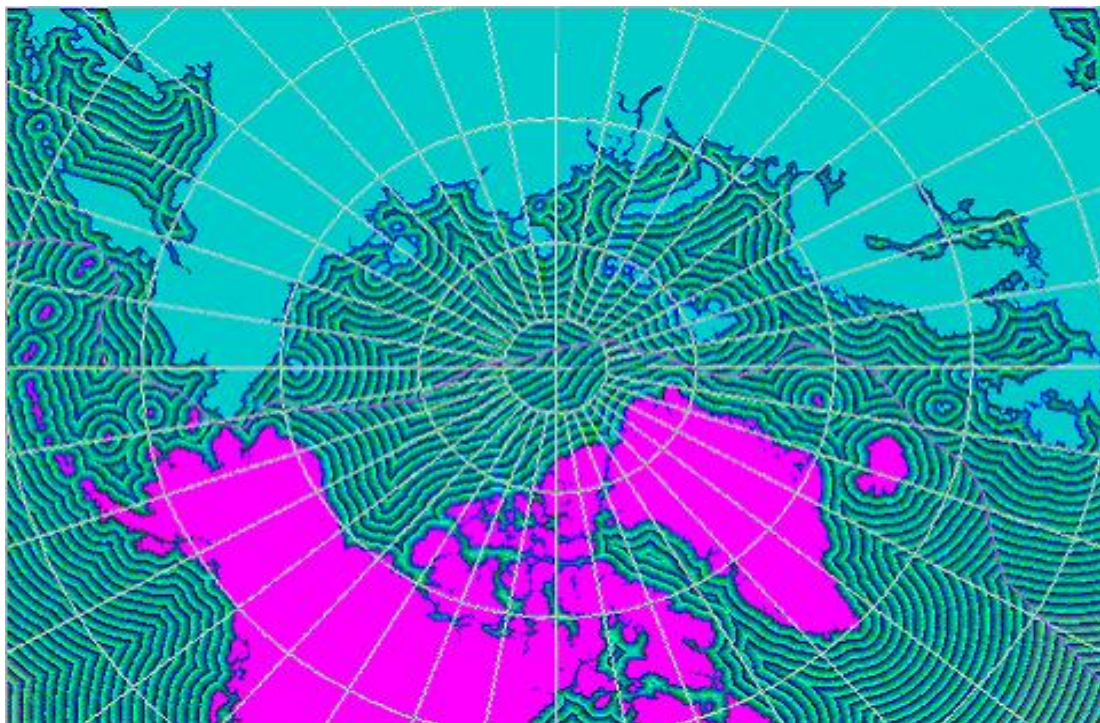
Expression and organization of spatial data are the foundation in Geoinformation science. Vector model has been widly used in E-space. But in practical earth space, complexity can not be expressed easily by vector model, instead, grid model has the maximum advantage.

disposal and analysis of spatial data

Nowadays, most disposals and analyses of spatial data aim only at E-space. We should pay more attention to the non-E-space.

4. disposal and analysis of spatial data

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