

# A NEW APPROACH FOR DESIGNING ORTHOPHANIC WORLD MAPS

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## ABSTRACT

In order to produce a world map on which the continents closely resemble their appearance on the globe one must make sure that both the shape and the relative size of the individual land masses are well represented. In this paper a method is proposed to quantify the shape distortion and the relative distortion of area on a map projection. A new approach is suggested for producing so-called balanced map projections, which have an equal amount of area and shape distortion. Balanced map projections occupy a different position in distortion space than map projections that are commonly used for world maps. While, according to the calibration method that is applied, balanced map projections have an equal distortion of area and shape, popular map projections like the Winkel-Tripel and the Robinson projection have a shape distortion that is much higher than the distortion of area.

## INTRODUCTION

From the early days of map making to the present time, the challenge of portraying the round Earth in two dimensions without introducing too much distortion has attracted the attention of geographers, physicists, astronomers, mathematicians and mapmakers. Until the 17<sup>th</sup> century most map projections proposed were based on simple geometric construction methods. With the development of the calculus, first applied to map projections by J.H. Lambert in his work of 1772, cartographers and mathematicians were given the necessary tools for developing map projections with interesting properties, the most important of these being the preservation of angles and area. From then on, the construction of graticules fulfilling certain geometric and distortion-related properties became an interesting mathematical exercise. For world maps in particular, developing map projections with special properties (e.g. no distortion of area, no distortion of angles, equidistance), starting from a number of assumptions with respect to the shape of the parallels and the meridians and the representation of the pole (point, straight line, curved line), led to a multitude of new graticules (Snyder and Voxland 1988, Canters and Declair 1989). While from a mathematical perspective many of these map projections are ingenious constructions, they are not necessarily interesting choices from a cartographic point of view. Indeed, visual assessment and mathematical analysis of the distortion pattern of map projections specifically developed for portraying the Earth as a whole shows that most of these projections introduce excessive shape distortion in some parts of the graticule or do not respect the relative size of the major land masses.

It was dissatisfaction with the overall appearance of the land masses on existing map projections that made Arthur Robinson decide in 1963 to develop a map projection of his own, which is known as the Robinson projection. Based upon a request by the Rand McNally Company for a simple and straightforward graticule for general-purpose world maps, suitable for readers of all ages, Robinson decided in favour of a pseudocylindrical projection with a pole line more than half the length of the equator, a ratio of the axes (equator to central meridian) not above 2:1, and parallels equally divided by the meridians. However, instead of proceeding in the traditional way, i.e. by developing map projection formulas starting from a mathematical description of the meridional curves (straight, circular, elliptic, parabolic, sinusoidal,...), he took a rather innovative approach. Starting with an arbitrary projection of the requested type, Robinson gradually adjusted the length and the spacing of the parallels, each time drawing a new graticule, plotting the continents, and judging the result. He repeated this process until it became obvious to him that further adjustment would produce no further improvement in the portrayal of the continents. As Robinson states "...the approach is essentially artistic in that the resulting projection is an interpretation distilled from the experience of the author" (Robinson 1974). Originally Robinson called his projection the *Orthophanic* (right appearing), implying that, according to his judgment, the projection avoids excessive distortion and produces an image of the continents that closely resembles their appearance on the globe.

Robinson's most important argument against the common approach to map projection development was that by starting from mathematical formulas that describe the general appearance of the graticule, and then adjusting a few parameter values (e.g. the value of the standard latitude, the length of the pole line or the ratio of the axes) to reduce distortion in

some parts of the map, the possibilities to alter the appearance of the continents are limited. To be freed from the restrictions imposed by the use of mathematical formulas, he opted for an empirical, non-formula approach. This offered him the flexibility needed to attain his goal, i.e. producing a map projection that portrays the continents in a way that deviates as little as possible from how we see the continents on the globe. The obvious advantage of this approach is that the map projection graticule, because of the way it is constructed, is aesthetically pleasing. On the other hand, there are also some disadvantages. Because the procedure for producing the graticule is based on personal judgement, the approach is highly subjective. The method is not repeatable and does not lend itself to a routine production of world maps with other geometric properties or orientation. Moreover, because projections of this kind have no mathematical formulas, their graticules cannot be easily reproduced. Accurate mapping of data in these projections, or analysis of their distortion properties, requires the development of closely fitting empirical formulas or the use of exact interpolators (Ipbuker 2004).

The Robinson projection dates from a time when computers were not yet commonly used in cartographic research. Since the 70s, however, much work has been published on the development of map projections with low distortion, using numerical techniques. Several studies have addressed the problem of developing new projections for world maps with low overall distortion (Peters 1978, Canters 1989, Laskowski 1991). The approach taken in these studies is to start with a set of map projection equations that contains a relatively large number of parameters, so that by changing the parameter values the graticule of the projection can be modified in a flexible way. Next to a general set of equations, a distortion measure is defined that quantifies the total amount of distortion for the region to be mapped (for a world map the region to be mapped usually corresponds with the continental area). Then, by applying a suitable optimization technique, a set of parameter values is determined that minimizes the value of the distortion measure.

Compared to the empirical approach outlined above, numerical reduction of distortion, starting from a set of general transformation formulas, has a number of advantages. Because optimization of distortion is an automated process, it is relatively easy to experiment with various constraints and to produce low error world maps with different graticule geometries (e.g. straight or curved parallels, pole is point or pole is line, ...), special distortion properties (equal-area graticules, true scale along equator and/or central meridian,...) and orientations (alternative positions of the central meridian, transverse or oblique aspects). The numerical approach is also objective in the sense that repeating the experiment under exactly the same conditions will reproduce the same graticule.

A major difficulty with the numerical approach, however, is the definition of a proper measure to quantify overall distortion. Over the years a multitude of distortion measures and algorithms for calculating an average distortion value for a graticule have been proposed. Some of these measures focus on one type of distortion (distortion of scale, area, angles, azimuth, distance, shape,...), while other measures assess the combined effect of two, three or even more aspects of distortion. Using different measures for minimizing map projection distortion produces low-error graticules that may look very different (Laskowski 1998). An obvious question therefore is if it would be possible to define a suitable distortion measure for numerically producing *orthophanic* map projections. Just like the Robinson projection, these projections should depict the major landmasses in a way that does not deviate too much from how we perceive them on the globe. In an attempt to attain this goal, this paper presents a new distortion measure that reduces the distortion of both the shape and the relative size of the continents. The distortion measure will be used to produce low-error graticules which, according to the way they are developed, can be considered *orthophanic* map projections.

## DEFINING A SUITABLE MEASURE OF DISTORTION

Recently, Canters (2002) proposed a method to quantify the relative distortion of area and the distortion of shape for map regions of finite size, hereby acknowledging that respecting the relative proportions of the continents as well as properly representing their shape are the two most important qualities we expect from a map projection that is intended to be used for general-purpose world maps. The method is based on the calculation of an average value of the relative area distortion and the distortion of shape for 1000 spherical circles of varying size (circular radius  $\leq 30^\circ$ ), which are randomly generated over the continental area. For each spherical circle 16 positions along its perimeter are calculated, corresponding to azimuthal angles of  $0^\circ, 22.5^\circ, 45^\circ, \dots$ , defined from the centre of the circle. By projecting each position along the circle's outline in the plane a polygon with 16 vertices is obtained, approximating the projected image of the circle in the plane. To calculate the average distortion of area  $E_A$  for the random set of circles the following distortion measure is proposed:

$$E_A = \frac{1}{m} \sum_{i=1}^m \frac{|S_i - S'_i|}{|S_i + S'_i|} \quad (1)$$

with  $S_i$  the area of a circle  $i$  on the globe,  $S'_i$  the area of the polygon approximating the projected circle in the plane, and  $m$  the total number of circles ( $m=1000$ ). The proposed measure has the property that only the relative distortion of area is taken into account, not the absolute difference in area, so that small and large circles equally contribute to the average distortion value. The distortion calculated for each circle is also equal for enlargements and reductions by the same factor, meaning for example that in the calculation of the average distortion of area equal weight is given to circles that are doubled in size and to circles that are reduced to half of their original size. The above properties, however, do not guarantee scale independency, meaning that if one would change the dimension of a projection's graticule by multiplying all coordinates by the same factor the average distortion value would not remain the same. If it is strictly the intention to use the average distortion of area for evaluating how well relative differences in size between the various land masses are maintained, then the distortion measure should be independent from scale. A simple way to accomplish this is by introducing a scale factor  $k_0$  in the transformation formulas of the projection and scale the graticule of the projection until the value of  $E_A$  is minimized. That way the values of  $E_A$  obtained for different projections will only account for relative distortions of area, and not for differences in scale. The minimum value of  $E_A$  can be converted to an area scale factor  $K_A$  that represents the enlargement of area that corresponds to that minimum value:

$$K_A = \frac{1 + E_A}{1 - E_A} \quad (2)$$

To measure the distortion of shape, Canters (2002) proposes a measure that is similar to the index of Boyce and Clark (1964), which describes an object's departure from the circular shape. After the spherical circle has been projected in the plane, map distances between the projected centre and each of the 16 peripheral points (see above) are calculated. Then the proportion of each radial distance with respect to the sum of all 16 distances is calculated and subtracted from the proportion each radial distance would represent if the circular shape would have been retained (1/16). The absolute values of these differences are then summed to obtain the value of the shape index for one projected circle. The value of the index becomes zero if the circular shape is perfectly maintained and increases as the shape becomes less compact. The index has the advantage of not being influenced by size and orientation. Averaging the value of the index for a large number of spherical circles, randomly generated over the land masses, provides an estimate of the overall amount of shape distortion  $E_S$  present in the map projection's graticule.

	$K_A$	$E_S$	$E_{A,c}$	$E_{S,c}$	$E$
Winkel-Tripel	1.159	0.098	0.194	0.444	0.638
Robinson	1.150	0.101	0.183	0.466	0.649
Kavraysky VII	1.209	0.093	0.255	0.409	0.664
Aitoff-Wagner	1.188	0.098	0.229	0.444	0.673
Eckert IV	1.000	0.133	0.000	0.694	0.694
Wagner VI	1.209	0.099	0.255	0.452	0.707
Hammer-Wagner	1.000	0.139	0.000	0.737	0.737
Plate Carrée	1.323	0.088	0.393	0.373	0.766
Mollweide	1.000	0.151	0.000	0.822	0.822
Miller I	1.606	0.050	0.738	0.103	0.841
Aitoff	1.098	0.137	0.119	0.722	0.841
Hammer-Aitoff	1.000	0.155	0.000	0.850	0.850
Sinusoidal	1.000	0.176	0.000	1.000	1.000

Table 1. Average area scale factor ( $K_A$ ), distortion of shape ( $E_S$ ), calibrated values for area distortion ( $E_{A,c}$ ) and shape distortion ( $E_{S,c}$ ), and total distortion value ( $E$ ) for well-known map projections

Table 1 lists the average area scale factor  $K_A$  and the average distortion of shape  $E_S$  for a selection of well-known map projections that are frequently used for world maps. To be able to compare the relative distortion of area for different map projections, the scale of the graticule of each projection was adjusted by minimizing the value of  $E_A$  from which the value of  $K_A$  is derived (see above). As could be expected, the distortion of shape is inversely related to the relative

distortion of area. Shape distortion is most prominent on equal-area projections and projections with low distortion of area. The highest value for shape distortion is obtained for the sinusoidal projection. As shape distortion decreases the relative distortion of area grows, reaching its highest value for the Miller I.

Because orthographic world maps should have as little as possible of both types of distortion, the most logical way to develop a projection for producing such maps is to define a distortion measure that quantifies the combined effect of area and shape distortion, giving equal weight to both, and then try to obtain a graticule that has the lowest possible value for this measure. A difficulty in defining such a combined distortion measure is that the relative distortion of area  $E_A$  (or the corresponding area scale factor  $K_A$ ) and the distortion of shape  $E_S$  are not expressed in the same units. Hence one cannot simply calculate the sum or the average of the values obtained for  $E_A$  (or  $K_A$ ) and  $E_S$  to assess the combined impact of both aspects of distortion. To solve this problem of unequal units we propose to map the values obtained for  $K_A$  and  $E_S$  on the [0,1] interval by specifying minimum and maximum values for both  $K_A$  and  $E_S$ , mapping these values on the endpoints of the interval, and then performing simple linear rescaling based on these values:

$$E_{A,c} = \frac{1}{K_{A,max} - K_{A,min}} (K_A - K_{A,min}) \quad (3)$$

$$E_{S,c} = \frac{1}{E_{S,max} - E_{S,min}} (E_S - E_{S,min}) \quad (4)$$

Because there is no relative distortion of area on an equal-area projection, the minimum value for the area scale factor  $K_{A,min}$  is equal to one. Defining a maximum value for the distortion of area is less straightforward, since theoretically there is no upper limit to the relative distortion of area that may occur. While we could have used the value of the Miller I as a reference value (the Miller I has the highest relative distortion of area of all standard map projections listed in table 1), this would be a somewhat arbitrary choice, as there may be other projections not included in table 1 that have a higher relative distortion of area. However, because an inverse relationship is observed between distortion of shape and relative distortion of area, one may assume that a maximum distortion of area will occur for graticules that have a minimum distortion of shape. Using the measure for shape distortion  $E_S$  defined above, we therefore produced new graticules with the least possible amount of shape distortion for different projection classes, using the optimization approach described in the next section of this paper, and then calculated the relative distortion of area for each of these graticules. Doing so, a maximum value for  $K_A$  of 1.8211 was obtained for a cylindrical projection with the least possible distortion of shape (not shown here). This value is substantially higher than the value obtained for Miller I and was defined as the upper limit for the area scale factor  $K_{A,max}$ .

Because shape distortion cannot be avoided, the minimum value for shape distortion  $E_{S,min}$  that can be attained will differ from zero. Reducing shape distortion without putting unnecessary constraints on the shape of the meridians and the parallels produces a graticule with  $E_S = 0.0356$  (not shown here). This value was used as a lower bound for the distortion of shape. The highest values for shape distortion are obtained for equal-area projections. Because many equal-area projections exist and their clearly is no upper limit to the distortion of shape that may occur, we decided to put  $E_{S,max} = 0.1760$ . This value corresponds to the distortion of shape for the sinusoidal projection which, because of the nature of its meridians, has the highest distortion of shape of all the equal-area projections included in table 1. While, theoretically, it is possible to define equal-area projections with a higher distortion of shape, such projections would be useless because of the high amount of distortion. Therefore the sinusoidal projection will be considered as the extreme case bounding one end of the distortion continuum, with a value for  $E_{A,C}$  equal to zero and a value for  $E_{S,C}$  equal to one.

By rescaling the values obtained for area and shape distortion using equations (3) and (4), all projections for world maps will occupy a unique position in a 2-dimensional normalized distortion space, with distortion values  $E_{A,C}$  and  $E_{S,C}$  ranging from 0 to 1 along both axes. By adding together the two calibrated distortion values  $E_{A,C}$  and  $E_{S,C}$  the combined effect of shape and area distortion can be assessed for any arbitrary projection:

$$E = E_{A,c} + E_{S,c} \quad (5)$$

As can be seen from table 1, the lowest values for  $E$  are obtained for the Winkel-Tripel projection (with standard parallels at 40°) and for the Robinson projection, which both combine a low relative distortion of area with a moderate

distortion of shape. Projections closer to both extremes of the distortion continuum (maximum distortion of shape or maximum distortion of area) have a higher total distortion value. The fact that both the Winkel-Tripel and the Robinson projection have the best overall score indicates that the distortion measure which we have defined succeeds in producing low distortion values for what presently are two of the most frequently used projections for world maps. The success of these two projections is definitely related to the fact that the portrayal of the land masses on these projections is perceived as not deviating too much from the way the continents are portrayed on the globe. The question at this point is whether the proposed distortion measure  $E$  allows us to produce graticules with possibly even lower overall distortion values and with a properly balanced distortion of shape and area, and, if so, in what ways these graticules will differ from well-known projections like the Robinson and the Winkel-Tripel.

## PRODUCING GRATICULES WITH LOW DISTORTION FOR WORLD MAPS

As said before, the most obvious approach to derive new graticules with low distortion is to start with a set of general map projection equations that offers a high level of flexibility for changing the appearance of the graticule by adjusting the value of a number of transformation parameters, and to search for an optimal combination of parameter values that minimizes distortion according to the distortion measure that has been selected. An interesting option is to make use of power series. In its most general form, assuming a unit radius for the generating globe, the relationship between map projection coordinates  $x$ ,  $y$  and geographical longitude  $\lambda$  and latitude  $\phi$  can be expressed by the following two polynomials:

$$x = \sum_{i=0}^n \sum_{j=0}^{n-i} C_{ij} \lambda^i \phi^j \quad (6)$$

$$y = \sum_{i=0}^n \sum_{j=0}^{n-i} C'_{ij} \lambda^i \phi^j \quad (7)$$

with  $C_{ij}$  and  $C'_{ij}$  the polynomial coefficients defining the properties of the graticule. The order of the polynomials  $n$  will determine the flexibility of the transformation. Canters (1989) used the fifth-order version of the above series for developing new projections for world maps with a minimum distortion of distance (see also Canters 2002, p. 187). Laskowski (1991) used the same series for developing a map projection with low error by minimizing the value of a distortion measure that assesses the combined impact of three different types of distortion. He also made use of the series to produce low-error graticules for a large number of alternative distortion measures (Laskowski 1998). In the present study the above series will be used to produce graticules with low distortion of shape and area, using the distortion measure  $E$  defined above as the objective function for optimizing the value of the polynomial coefficients.

By putting appropriate constraints on the values of some of the coefficients a multitude of map projection graticules can be derived, from highly complex graticules to graticules that belong to traditional map projection classes. Working with fifth-order polynomials, and assuming that the graticule is symmetric about the central meridian and the equator, equations (6) and (7) reduce to:

$$x = C_{10} \lambda + C_{30} \lambda^3 + C_{12} \lambda \phi^2 + C_{50} \lambda^5 + C_{32} \lambda^3 \phi^2 + C_{14} \lambda \phi^4 \quad (8)$$

$$y = C'_{01} \phi + C'_{21} \lambda^2 \phi + C'_{03} \phi^3 + C'_{41} \lambda^4 \phi + C'_{23} \lambda^2 \phi^3 + C'_{05} \phi^5 \quad (9)$$

If we also assume that the equator is equally divided by the meridians, which is the case for most projections used for world maps, then equation (8) further reduces to:

$$x = C_{10} \lambda + C_{12} \lambda \phi^2 + C_{32} \lambda^3 \phi^2 + C_{14} \lambda \phi^4 \quad (10)$$

leaving us ten parameters for graticule adjustment. A projection of this type will generally have curved meridians and curved parallels. By removing some of the parameters in equations (9) and (10) even less complex graticules can be obtained. For example, if the  $y$ -coordinate is made a function of the latitude only (by putting  $C'_{21}$ ,  $C'_{41}$  and  $C'_{23}$  equal to zero), a projection with straight parallels is obtained (a so-called pseudocylindrical projection). If, in addition, the  $x$ -coordinate is made a function of the longitude only (by putting  $C_{12}$ ,  $C_{32}$  and  $C_{14}$  equal to zero), then only four parameters will be left, and the formulas will describe a cylindrical projection with adjustable spacing of the parallels. Other characteristics of the graticule (spacing of the parallels, ratio of the axes, representation of the pole,...) can be controlled by putting additional parameter values equal to zero, by defining a functional relationship between two or more parameters, or by slightly modifying the original formulas (for more details, see Canters 2002, p. 138).

In this study the polynomial model with symmetry about both the central meridian and the equator, and with the equator being equally divided by the meridians (equations (9) and (10)) was used as a starting point for developing graticules with low distortion of shape (to produce reference distortion values for calibration, see above) as well as graticules with moderate distortion of shape and area. To produce a set of polynomial coefficient values that minimizes the value of the distortion function ( $E_S$  for projections with low distortion of shape,  $E$  for projections with balanced distortion properties) an iterative algorithm, known as the simplex method, was applied. The algorithm moves through the "distortion landscape" in a  $k$ -dimensional factor space defined by the  $k$  polynomial coefficients in the set of equations in search of the location (combination of parameter values) that produces the lowest value for the distortion function. Low-error graticules were developed for different classes of projections, by putting appropriate constraints on the coefficients values in the polynomial transformation model. Some examples of low-error graticules obtained by reducing both shape and area distortion will now be discussed.

## MAP PROJECTIONS WITH BALANCED DISTORTION OF AREA AND SHAPE

Based on the general polynomial transformation model, defined by equations (9) and (10), different graticules were produced by minimizing the distortion function  $E$  over the continental area (Antarctica not included). Each graticule was obtained by applying a unique set of constraints, leading to low-error map projections with different geometric properties. One of the most important observations was that for most of the graticules that were obtained by minimizing the value of  $E$  the two components of distortion  $E_{A,C}$  and  $E_{S,C}$  did not equally contribute to the value of the distortion function.

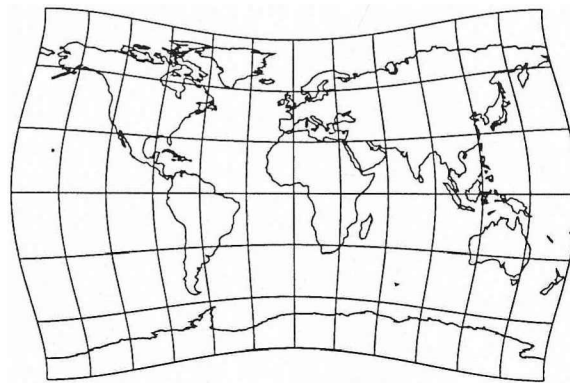


Figure 1. Low-error map projection with curved meridians, equally spaced along the equator, and curved parallels, equally spaced along the central meridian ( $E_{S,c} = 0.225$ ,  $E_{A,c} = 0.373$ ,  $E = 0.598$ )

Figure 1 shows an example of a low-error graticule that was obtained by minimizing the value of  $E$  for a transformation model with curved meridians and curved parallels, equally spaced along the equator and the central meridian. As one can see, the shape distortion for this graticule contributes much less to the overall distortion value than the relative distortion of area (0.225 against 0.373). The low distortion of shape is clearly linked to the curvature of the meridians, which is much less pronounced than on other map projections of this class we are familiar with. Indeed, if we compare the optimized graticule with the Winkel-Tripel projection, which has similar geometric properties, we observe a strong difference in the curvature of the meridians. By comparing the distortion values for both projections it becomes clear that this difference in the curvature of the meridians has a strong impact on the balance between area and shape distortion. While the total distortion value  $E$  for the Winkel-Tripel projection is not much higher than for the optimized

graticule (0.638 for the Winkel-Tripel, 0.598 for the optimized graticule), the shape distortion for the Winkel-Tripel is twice as high. Apparently both projections occupy different positions in the normalized distortion space. While the Winkel-Tripel projection belongs to the group of map projections on which the distortion of shape is most prominent, the optimized graticule is part of the group of map projections that has more distortion of area.

The fact that the optimized graticule in figure 1 and the Winkel-Tripel projection have overall distortion values that do not differ much suggests that it should be possible to develop series of low-error graticules with similar geometric properties and with about the same amount of overall distortion, yet with each graticule in the series corresponding to a different balance between area and shape distortion. Low-error graticules for which the contributions of area distortion and shape distortion are equal would then represent a special case in such a series and might be referred to as *projections with balanced distortion of area and shape* or *balanced projections*. To produce low-error graticules of this type we adopted a two-step approach. After first having generated a low-error graticule by minimizing the overall distortion value  $E$ , we gradually adjusted the geometry of the optimized graticule by reducing the difference between the calibrated values for area distortion  $E_{A,C}$  and shape distortion  $E_{S,C}$  until the contributions of both types of distortion become the same. Both steps of the procedure are accomplished by means of the simplex method. However, in the second step precautions have to be taken to ensure that, while the difference between  $E_{A,C}$  and  $E_{S,C}$  is minimized, the value of  $E$  obtained in the first step of the procedure is not allowed to increase by more than a small, user specified tolerance.

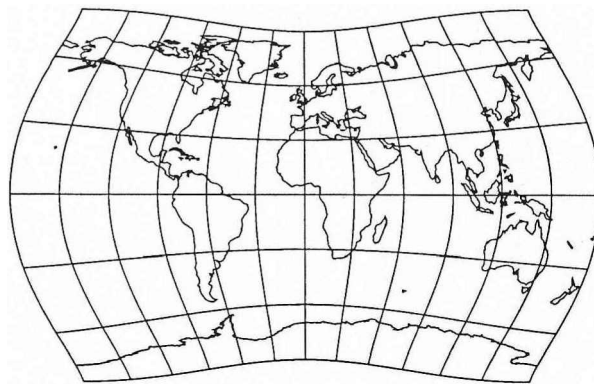


Figure 2. Low-error balanced map projection with curved meridians, equally spaced along the equator, and curved parallels, equally spaced along the central meridian ( $E_{S,c} = 0.320$ ,  $E_{A,c} = 0.320$ ,  $E = 0.640$ )

Figure 2 shows an example of a balanced map projection that was obtained by starting from the optimized graticule in figure 1, and reducing the difference between area and shape distortion until both types of distortion are in balance. A comparison of figures 1 and 2 shows that the reduced distortion of area is accomplished through a stronger curvature of the meridians, which inevitably leads to a higher distortion of shape. On the whole the adjusted graticule is more appealing than the original one, because the increased curvature of the meridians slightly reduces the east-west stretching in the higher latitudes. However, the graticule still substantially differs from the Winkel-Tripel projection which, through a further increase of the curvature of the meridians, reduces the stretching of the polar areas and the area distortion at the expense of an increase in the overall distortion of shape

Of course, the method for producing balanced map projections that has just been proposed can also be used to generate low-error graticules with other geometric properties. Figure 3 shows an example of a low-error balanced map projection of the pseudocylindrical type with a pole line half as long as the equator, and with the parallels equally spaced along the central meridian. The projection is similar to Kavraisky VII, which also has straight, equally spaced parallels and a pole line half the length of the equator. What distinguishes the balanced projection from Kavraisky VII, however, is again the curvature of the meridians. Indeed, while on Kavraisky VII, and on most other pseudocylindrical map projections we are familiar with, the curvature of the meridians gradually increases from the equator towards the poles, this is clearly not the case for the balanced projection, where in the lower latitudes the meridians are almost straight lines. Just like in the previous example, it is this reduced curvature of the meridians that leads to less overall distortion of shape. While, according to the calibration method we have applied, the optimized projection has an equal distortion of area and shape, for Kavraisky's seventh projection the shape distortion is much higher than the relative distortion of area. A similar disparity between area and shape distortion was noticed for the Winkel-Tripel projection (see above) and is also observed for other non-equal-area projections of the pseudocylindrical and polyconic class listed in table 1 (Robinson,

Aitoff-Wagner, Wagner VI). This clearly illustrates that the balanced map projections proposed in this paper occupy a different position in distortion space than the map projections that are commonly used for world maps.

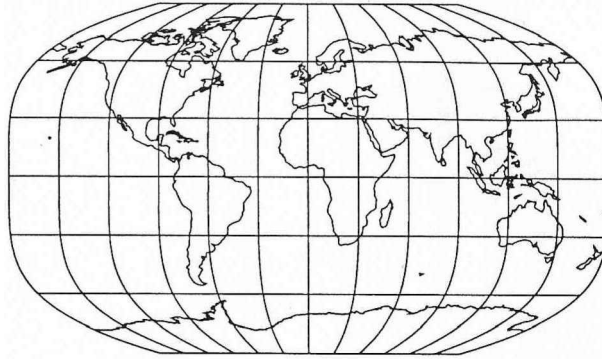


Figure 3. Low-error balanced map projection with curved meridians, equally spaced along the equator, straight parallels, equally spaced along the central meridian, and with a pole line half the length of the equator ( $E_{S,c} = 0.307$ ,  $E_{A,c} = 0.307$ ,  $E = 0.614$ )

## CONCLUSIONS

In order to produce a world map on which the continents closely resemble their appearance on the globe one must make sure that both the shape and the relative size of the individual land masses are well represented. In this paper a new distortion measure was proposed to quantify the joint contribution of shape distortion and relative distortion of area. An optimization approach has been suggested for producing map projections with an equal amount of area and shape distortion. The major difference between these so-called balanced map projections and other map projections with intermediate distortion properties lies in the curvature of the meridians, which for balanced projections proves to be much less pronounced. This difference in the curvature of the meridians explains why commonly used projections for world maps occupy a different position in distortion space, with, according to the calibration method we have applied, a distortion of shape that is much higher than the distortion of area.

What projection is to be preferred is partly a matter of taste, and cannot be based purely on quantitative assessment. The advantage of the numerical approach proposed in this paper, however, lies in its flexibility. First of all, the approach makes it easy to experiment with different geometric constraints that have a major influence on the appearance of the low-error graticule that is obtained. The mechanism for balancing area and shape distortion that is suggested can easily be modified to produce low-error graticules that give more weight to one distortion component than to the other. As such, the method might also be applied to generate low-error graticules that occupy a position in distortion space close to the position of the projections that are currently used for world maps. Investigating the impact of changing the balance between area and shape distortion on the appearance of the low-error graticules that are obtained will be the subject of future research.

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## **BIOGRAPHY**

Frank Canters obtained the degree of Doctor of Science from the Vrije Universiteit Brussel (VUB) in 1999 with a dissertation on small-scale map projection design. Since 2002 he is a lecturer in cartography, GIS and spatial analysis in the Department of Geography of VUB and is leading the Cartography and GIS research group. He is also guest professor at the University of Ghent where he teaches a course on map projections and coordinate systems since 2001. His research interests are map projection design, modelling of uncertainty in spatial data, land-use/land-cover mapping from high-resolution satellite imagery, and multi-source/multi-resolution modelling approaches in remote sensing and GIS.

Frank Canters is the author of two books on map projections. In 1989, together with Hugo Declair, he published *The World in Perspective, A Directory of World Map Projections*, a comprehensive survey of the distortion characteristics of well known and less well known projections for world maps. More recently he published *Small-scale Map Projection Design*, which appeared in the Research Monographs in Geographic Information Systems series by Taylor & Francis in 2002.

Frank Canters was vice chair of the Belgian Committee of Cartography and GIS from 1997 till 2002. Since 2003 he is vice chair of the ICA Commission on Map Projections.