MULTI MAP PROJECTION IN MODERN CARTOGRAPHY

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ABSTRACT

The requirement of seamless spatial data integration has driven the needs of developing multi map projection (MMP) in modern cartography. MMP is defined as an intelligent map projection selection so that the distortions are minimized during map rendering. By visual navigation from equator to pole, different projections such as Transverse Mercator, Lambert Conic Conformal, or Azimuthal may be applied in view window in order to minimize distortions.

This paper presents the concepts of MMP, and scientifically evaluation of projection selection parameters of projection selection during the visualization. Also elaborate the value of such a system in cartographic communication. It is expected that a perfect implementation of this concept, would help Spatial Data Infrastructure to overcome their map projection challenges.

1. INTRODUCTION

Inconsistent map projections can be a problem in many areas of cartography. Using GIS systems by non-experienced users might lead to an unconsidered choice of projection and subsequently the message of the map attempting to communicate can significantly be devaluated. Geographic databases are not analogue paper anymore and in digital era conditions change per user zoom operator [Helali, *et al*, 2003].

In the past, hundreds of projections have been developed to best solve the problem of representing the spherical earth on a flat map. We are in an era when almost everyone has the tools to create a "map". Although the presentation of construction methods may not be necessary any longer, the characteristics and appropriate use of different projections are still of great importance to cartographers.

Different expert systems have been developed in the past decades to help user in intelligent selection, EMPSS by Snyder (1989), MaPKBS by Nyerges and Jankowski (1989), and MaPSS by Kessler (1991). All of them;

- Running in static conditions whose results would be used for paper maps.
- There are no exact separation in effecting parameters like scale rang and geographic location.
- Prepared for a limited map projection, farther country applied map projections and old map projections are not considered.
- Information gathered by communicating with user, which increase user interactivity.
- Almost their differences are in their selection knowledge bases.

These characters cased theses systems would be used just in academic researches. Rather than automatic map projection selection in digital maps is required.

Multi Map Projection (MMP) is defined as an intelligent selection of a map projection so that the distortions are minimized during map rendering. By visual navigation from equator to pole, different projections such as Transverse Mercator, Lambert Conic Conformal, or Azimuthal may be applied in view window to have a minimum distortions.

There could be three stages in MMP research agenda:

i) Transforming the view window conditions to impress parameters, which affecting selection process.
ii) Identification and modeling selection criteria of map projections properties, which result in map projection database.
iii) Develop an expert system for MMP selection process.

In the current research cover first two stages. The second section discusses the impressing parameters for map projection selection. Also, it elaborates the implemented methods to extract those parameters from view window. Third section covers the methods to model map projections new properties, which are geodesic and have been implemented for the first time. Forth section is about the techniques that used to identify existing map projection properties. Moreover, a brief elaboration of implemented system will come in section five. The last section will discuses about MMP benefits in map communication and its expected applications.

2. TRANSFORMING THE VIEW CONDITIONS TO IMPRESSING PARAMETERS

In a MMP system, view windows are equal to maps which users look on. Therefore it considers all map projections and models some geographic impressing parameters such as *view projection* impressing. By over viewing different selection schemes some of them appear: application, extent of map projection, geometric and general properties, Loxodromic, Azimuthal. Also this research adds some other properties for map projections thereby selection process would be done easier in all conditions. Map projection scale rang (Scale-min, Scale-max, compression method), map projection limits (*λ_min, λ_max, φ_min, φ_max*), Datum validation limits (*D_φ_min, D_φ_max, D_λ_min, D_λ_max*), time priority, and country in which map projection is applied are in this category requested to be modeled and identified.
As it has been mentioned, a MMP system obtains some conditions by using view window, other conditions (View application/orientation) which can't be quite intelligently determined are set as default and user can modify them with interface tools in order to use them in expert selection system.
Scale, location and country conditions are quantities as a change happens in view window. The system obtains the view window conditions if any change in visualization is done by user. They may include:

- View scale
- View position
- View Country
- View window shape

Approaching to these mentioned parameters from view properties is discussed as following.

2.1 View Scale

Map extent is one of the most effective conditions in map projection selection. A MMP system considers view extend as its scale on a view window. In current methods scale is considered as a ratio of a projected distance to the same geodesic length. However, it must be understood in which line. Regarding to map projections distortions, scale distortion is not the same on different directions. During the research a new accurate method has been developed to calculate scale for all lines. This method considers optimum scale as S in equation 1

$$S = \sqrt{\frac{A_v}{A_e}} \qquad \text{(eq. 1)}$$

Where A_v is view window area and A_e is area of projected view on datum.

Computing A_e on the ellipsoid have done by

As results indicate that as the number of liens to calculate the scale increase its average ($\bar{S}$) approaches to S in equation 1. In other word when n in equation 2 increase ($\bar{S}$) approaches to S.

$$\bar{S} = \frac{\sum_{1}^{n} S_i}{n} \qquad \text{(eq. 2)}$$

This means S could be the average of all scales computed by all lines. Further study is going on to prove the method by mathematics. Figure 1 shows view window parameters.

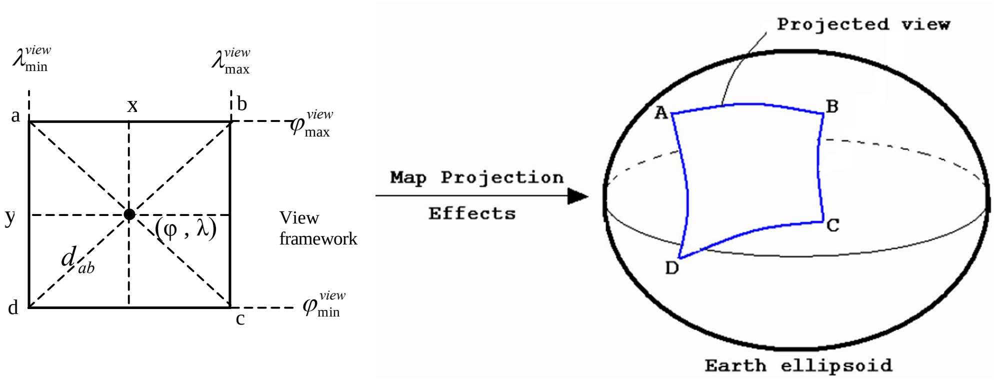


Figure 1, scale computation parameters.

Since these distances have different distortions, this method of scale computation is completely in dependent from measured distance.

2.2 View Position

Since large scale map projections are appropriate in specified geographic location (φ and λ), the view position are used as map geographic location. For each map projection valid range of latitude ($\varphi_{\min}^{projection}$, $\varphi_{\max}^{projection}$) and longitude ($\lambda_{\max}^{projection}$, $\lambda_{\min}^{projection}$) are calculated as it has been discussed on section 3.2. If view center's (φ and λ) are within this range (map projection validation boundary), the expert system would identify that map projection due to its range. Moreover, on whether view boundary's can be used instead of view center. Thereby, the projection having the most matching is recognized. Since the parameters are increasing, the chances to reach a unique map projection increase.

2.3 View Country

Due to this fact, some map projections are defined for specific countries, it would be more efficient to find map projection of each country located in view boundary. To detect the specific country, view center's coordinates must be tested through the polygons of countries. So if the view center located in a country have specific map projection; and scale condition is authenticable, country's map projection is identified. When there are more than one country in view window, the country with greatest area will be selected.

2.4 View Framework Shape

View window may have different shapes (rectangle, circle, ellipsoid…). Each of them has different specification, which makes them to be suitable for some map projections. As it is clear this suitability has relation with map projections outline. To keep the same symmetry in horizontal and vertical direction, this research consider window set as square instead of rectangle, although almost all monitors have rectangle screen.

3. MODELING SELECTION CRITERIA FOR EXISTING MAP PROJECTIONS PROPERTIES

As it is known each map projection is only suitable for a specific condition that such as an application, scale and geographic location. To consider all conditions in the selection process it is a need to clarify map projections parameters precise value for all of them.

3.1. Scale

When scale changes distortions (length, area and angle), in different parts of the map changes appear as well. So by decreasing scale (increasing map extend) the map mean distortion error would be increased in the view. To compare map projections distortions based on scale, Tissot's indicatrix would be used [Peters 1984, Canters 1989, Laskowski 1991]. To reach the view's mean distortion, Tissot's indicatrix on different map projections is calculated, for 100 homogeneity distributed points in view window. Moreover;

- This comparison must be stand-alone from the position of map projection center. In other word, the comparing map projections must have been defined at the same point.
- Also Tissot's indicatrix has not similar behavior in each map projection class. Therefore, it is needed to compare a map projection in the same class.

For choosing the optimum map projection to provide the least distortion error in the view window, two methods have been implemented:

- Defining scale range for each type of map projection class.
- Comparing the map projection distortion error at the same scale (extend) at run time.

3.1.1 Defining Scale Range

In this method, the MMP system is based on the predefined information, and recognizes the most appropriate map projection type (Cylindrical, Conical, Planer etc.) per view scale. So, scale range assigned for each map projection. To define scale range exactly, there is need to determine; first accurate scale independent from projection and location, and second, find out in which scale interval what type of map projection produces the least distortion in comparison with other types, then that type would be the first priority in that scale interval.
In case of disconformities of the map projection with other required terms, the system looks for a map projection with less priority.
Therefore, the mean distortion error in the view window has been calculated for different types and scales. We considered Map projection classes in three categories: conformal, equidistant and equal-area, then scale-distortion graph is drawn on each class. To get map projections distortion per each view scale (1/1000~ 1/200,000,000), 100 point in a 10*10 grid were used. The graphs will show us in each scale, map projection priority by their closeness to 1.

3.1.1.1 Scale Range in Conformal Projections

As the distortion indicator of conformal projections (Tissot's Ellipsoid) transforms to a circle and distortion error in a point is the same in all directions (figure 2), comparison parameter for each conformal projection would be the Tissot's circles area [Canters, 2002]. The more the Tissot's circle area gets close to π, the more the area distortion error decreases and that type of map projection would be more suitable, with regarding to this point, our problem in a conformal projection is area distortion.

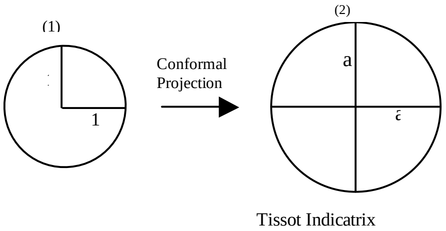


Figure 2, Tissot's indicatrix in conformal projections

$$\frac{S(2)}{S(1)} = \frac{\pi a^2}{\pi} = a^2 \qquad \text{(eq. 3)}$$

In conformal projections a^2 represents distortion error in each point so $\bar{t} = \bar{a}^2$:

$$\bar{t} = \sum_{i=1}^{100} t_i \Big/ 100 \qquad \text{(eq. 4)}$$

With t_i distortion on a point and $\bar{t}$ is mean distortion error on view.

- Distortion in cylindrical conformal projection (e.g Mercator) [Richardus,1972]:

$$a_i = 1 \big/ Cos\,\varphi_i \qquad \text{(eq. 5)}$$

Where φ_i is point geodesic latitude.

- Distortion in conical conformal projection (one standard parallel) [Richardus,1972]:

$$P_i = RCot\,\varphi_0 \left(\frac{tg(45° - \varphi_i/2)}{tg(45° - \varphi_0/2)} \right)^{Sin\,\varphi_0} \qquad \text{(eq. 6)}$$

$$a_i = \frac{P_i Sin\,\varphi_0}{RCos\,\varphi_i} \qquad \text{(eq. 7)}$$

Where R is radius of the earth and φ_0 is geodesic latitude of the contact parallel.

- Distortion in conical conformal projections (two standard parallel) [Richardus,1972]:

$$P_i = R\frac{Cos\varphi_1}{Sin\varphi_0} \left(\frac{tg(45° - \varphi_i/2)}{tg(45° - \varphi_1/2)} \right)^{Sin\varphi_0}$$

$$Sin\varphi_0 = \frac{Ln(Cos\varphi_1) - Ln(Cos\varphi_2)}{Ln(tg(45° - \varphi_1/2)) - Ln(tg(45° - \varphi_2/2))} \qquad \text{(eq. 8)}$$

Where φ_1 and φ_2 are geodesic latitude of the contact meridian

- Distortion in transverse cylindrical conformal projection (Transverse Mercator) [Richardus,1972]:

$$a_i = \frac{0.9996}{\sqrt{1 - (Cos\varphi_i . Sin(\lambda_i - \lambda_0))^2}} \qquad \text{(eq. 9)}$$

Where λ_0 is geodesic longitude of the contact parallel.

- Distortion in Planer Conformal Projection (Stereographic) [Richardus,1972]:

$$a_i = \frac{2tg(\varphi_0/2 - \varphi_i/2)}{Cos\,\varphi_i} \qquad \text{(eq. 10)}$$

When scale-distortion graphs had been produced for different classes of conformal projections, by overlaying graphs and studying their intersection point, scale ranges define exactly. The following figure shows scale-distortion graph for five types of conformal projections.

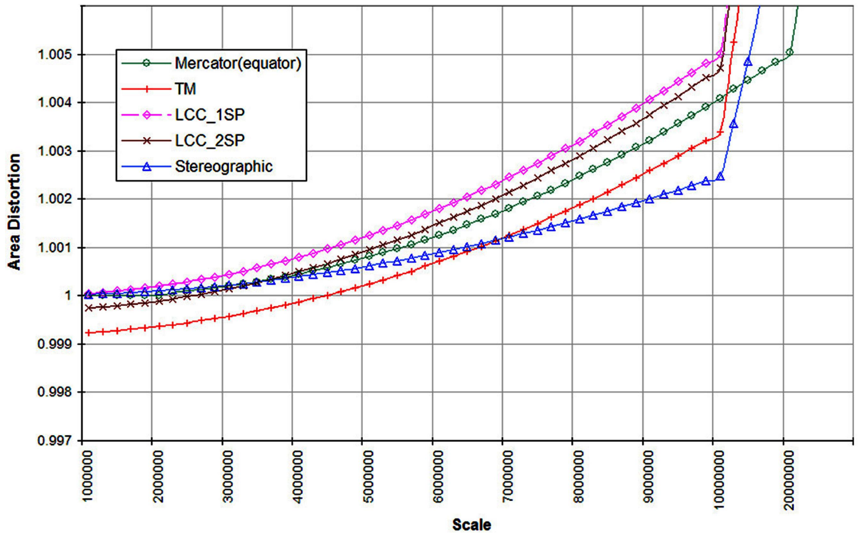


Figure 3, Scale- Distortion graphs in conformal projections

With considering the intersection points (the points which have the same distortion in two map projections), the priority order would be driven. Table 1 shows the eight scale range which was found out from Figure 3.

Table 1, Priority order of conformal projections in different scale ranges

Scale Range / Projection Type		Cylindrical	Reversed Cylindrical	Conical LCC_1SP	Conical LCC_2SP	Planar Stereographic
E+3	18E+5	1	5	3	4	2
18E+5	24E+5	1	5	4	2	3
24E+5	35E+5	2	5	4	1	3
35E+5	7E+6	3	1	5	4	2
7E+6	13E+6	3	2	5	4	1
13E+6	32E+6	1	3	5	4	2
32E+6	108E+6	2	3	5	4	1
108E+6	200E+6	5	2	3	4	1

3.1.1.2 Scale Range for Equidistant Projections

Equidistant projections have length distortion in the parallels direction [Kennedy, 2000]. So, distortion in the parallel direction (μ_p) is the best comparison parameter in equidistant projections. Therefore, a map projection type has priority when its scale factor in the parallel direction approaches to other in the view window. For this purpose, the average of μ_p is calculated in the view window (in a different scale), then a map projection type is selected that its average μ_p has the least difference to one in that scale.

- Distortion in azimuthal equidistant projection [Yang, *et al*. 2000]:

$$\mu_{P_i} = \frac{P_i}{RSinZ_i} = \frac{Zi}{SinZi} \qquad \text{(eq. 11)}$$

Where Z_i is Azimuthal angel

$$Z_i = Arc\cos(Sin\varphi_i.Sin\varphi_0 + Cos\varphi_i.Cos\varphi_0.Cos(\lambda_i - \lambda_0)) \qquad \text{(eq. 12)}$$

Where λ_0 and φ_0 is longitude and latitude of the central parallel and meridian

- Distortion in equidistant cylindrical projection [Yang, *et al*, 2000]:

$$\mu_{P_i} = \frac{C}{r_i} = \frac{R}{RCos\varphi_i} = \frac{1}{Cos\varphi_i} \qquad \text{(eq.13)}$$

Where C is radius of cylinder and r_i is radius in each meridian.

Figure 4 shows scale-distortion graphs of those two equidistant projections.

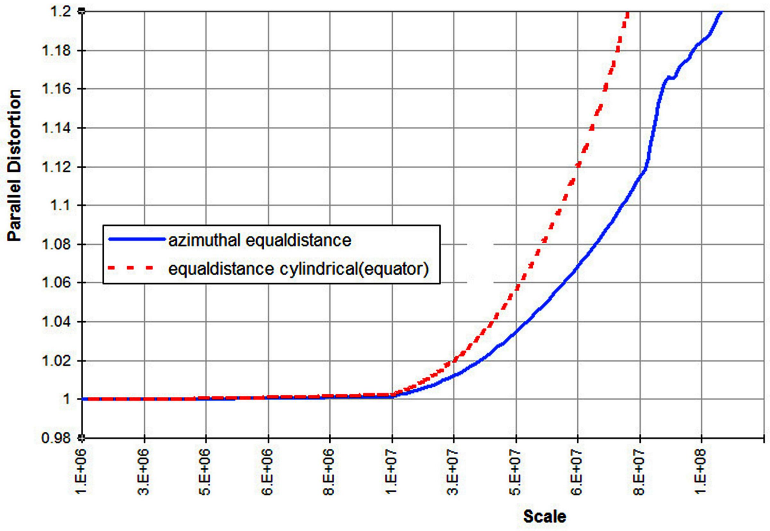


Figure 4, Scale-distortion graphs of Equidistant Projections

As the figure shows, it is clear that in the scales grater then 1/12,000,000 there is no much difference. But in the smaller scales azimuthal equidistant have minimum distortion error. Therefore in scales smaller then 1/12,000,000 it is suitable to use azimuthal equidistant map projections instead of cylindrical equidistant.

3.1.1.3 Scale Range for Equal-Area Projections

In equal-area Projections, since in all types of equal-area projections area of the Tissot's ellipsoid is equal to one; distortion in length is not same as all directions [Canters, 2002]. Therefore an error ellipsoid is produced for each point in which the direction of the long diagonal represents the direction of the maximum scale factor and the direction of the small diagonal shows the direction of the minimum scale factor. The criteria for selecting the best equal-area projection could be the Maximum Angular Distortion.

Regarding to, equal-area projections are rarely used in large scale maps and most of large scale maps have conformal projections which are approximately equidistant and equal-area like UTM; so changing a map projection due to changing the scale does not practically make sense.

3.1.2 Comparing the Map Projection Distortion Error

In this method the system searches for the map projection in the map projections database which has minimum distortion for view window at run time.

The method is same as previous one, but to reduce the computation time two techniques have been add. First, the candidate map projections have been selected by using other required terms (geodesic location, application…). Second, the mean distortion of each map projection is calculated in the view window by low density points. Mean distortion error of these points is attributed as the mean error of the whole view to the associated map projection. Finally, a map projection with least mean distortion error in the view window would be selected.

To reduce the number of candidate map projections, those which have other required terms are put in the selection set first. If there are multiple map projections of one type in the selection set (for example UTM zones), for reducing processing operations, distortion is calculated for just one of these map projections. In each type and class, a map projection on which calculations are done is a map projection which its central latitude and longitude are the closest in quantity to the ones of the center of the view window.

The advantage of this method is that we come to a unique selection of a map projection which produces the least distortion error, because these calculations are done directly on the current view. Also this method considers the position and the projection type, and there is no need to have scale priority in map projection properties database. The main defect of this method is the high computation process and accordingly a decrease in the speed of the system.

First technique would be fast and it is possible to have scale priority in map projection properties database. But, that priority is just in map projection classes and it is not possible to define all types in the same point. Therefore, this technique does not exactly separate map projection in scale criteria. Also the scale-distortion graphs depend to geodesic location of the view, which means in case a map projection is in higher priority, due to the remoteness of map projection's contact place with the view center a higher distortion error may be produced.

3.2. Geographic Boundary of Map Projections

Map projections are defined to project a particular geographic location using the datum information in which they have the least distortion errors. An ideal case to determine these boundaries is to use polygons. In this research rectangular polygon is consider and defined by its four limitation parameters (λ_min, λ_max, φ_min, φ_max). Some map projections

that are defined to apply in a specified country or region, in that case parameters are determined by their country boundaries.
Longitude (λ): In local map projections, there is more sensitivity to λ. This means that in large-scale (e.g.; 1:25000) λ has an affect on UTM zones and other zonal map projections are defined in a specific geographic location (φ and λ are the view center).
Latitude (φ): this parameter affects the map projections classes and defines northern or southern area. According to the fact that the defined datum of a map projection is used for modeling of geoeid in bounded area and for a specific country; φ and λ are sensitive to datum. For instance, UTM projection is defined for Australia with its own datum, φ is limited (beside λ limitation), and this limit is smaller than general UTM's φ limit.
Therefore, datum boundaries must be considered in selecting an MMP. In this study, all information relevant to validation boundary of a map projection and datum are collected according to reliable tables and references (e.g. Gänger-Bruckner 2002, POSC 1999, Agile Image 1999).

4. IDENTIFYING EXISTING MAP PROJECTIONS PROPERTIES

As it has mentioned, map projections have special properties that could be identify. This section shows how they identified. These parameters are general properties (conformality …), geometric properties (shape of the poles …), etc.

4.1. General Properties

Diverse map projections can be classified in three classes: conformal, equal-area, and equidistant. At implemented database, each map projection ranked in different cases: perfect, good, and week. Depends on reasoning mechanism ranking could be fuzzy. In fact, as it set these weighted values in database.

4.2. Geometric properties

Geometric properties are helpful in small scale map projection selection also these parameters mostly affected by user's application.

- Symmetry of graticules are considered in 5 categories, Radial symmetry, Single symmetry, Double symmetry, map projections having all 3 types of symmetry, and No symmetry.
- Space of graticules in this case categorized to 4 types: Unequal space, Equal space along central meridian, Equal space along equator, Equal space along central meridians and equators.
- Shape of graticules categorized to 4 types: Both meridians and parallels as curves, Meridians as straight lines and as curves, Parallels as straight lines and meridians as curves, both meridians and parallels as straight lines.
- Outline of the map considered in 5 categories: Circular, Rectangle, Oval, and others.
- Representation of the poles; in 2 categories
- Ratio of the axes; in 2 categories

4.3. Other Properties

- Time priority

During the past decade by introducing new precise datums some map projections are defined reiterative with respect to new datum. Therefore there are some maps in both old and new datum, but only one map projection with common properties. According to precision of the new datum in default situation it has priority, but there is capability to user to change time priority by considering map projection datum date.

- Map projection type

Each map projection is defined by means of a projection type and a specific method which there is correlation between applications. These classified in 14 types (e.g. Azimutal, Lambert Conformal Conic (1 standard parallel) , Cylindrical, etc.)

- Presentation of Loxodrome curves

This shows if each map projection preserves Loxodrome curves as straight line or not. Planar map projections have particular properties. Six classes are defined in this research (e.g. Orthodromic, Stereographic, Equidistant).

- Country map projections

Regarding to statement in map projection boundary, some map projection belong to a specified country. Since view position would be compared to detect the country, it should be mentioned in database.

- Application

Map projection's applications have relation with their general and geometric properties. It is clear this inter discipline should be lie on the selection knowledgebase. The map projections applications considered in more than 20 types.

5. IMPLEMENTED SYSTEM SPECIFICATION

By the adding new properties to map projections choosing a map projection in a MMP system in large scale representations would be easier, but we came to great challenges in prioritizing parameters in small scale maps. It has In the implemented system Canters (2002) suggested scheme modified and has been used as a knowledgebase. Figure 5 shows general process of developed MMP system of this study.

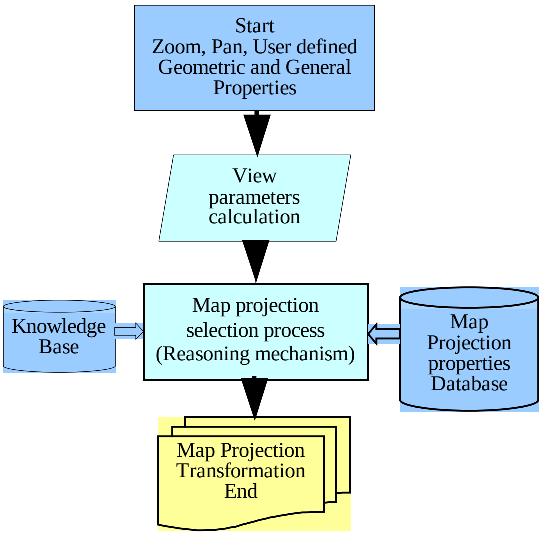
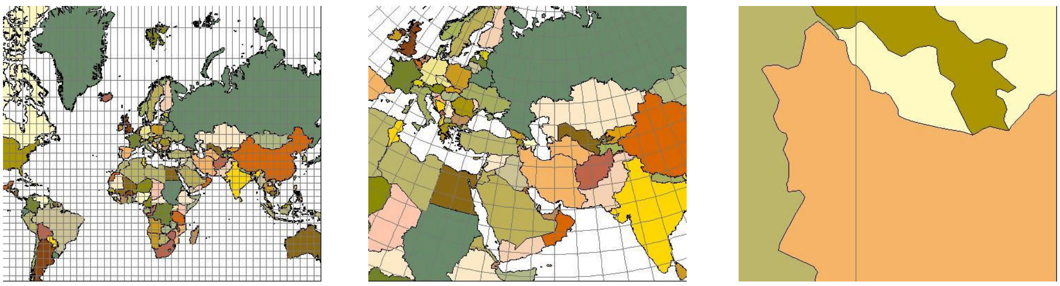


Figure 5, General process in MMP system

The process repeat as soon as a change is detected in size, location and other properties of view. When user changes the zoom or pan, the view parameters intelligently computed. This together with the general and geometric properties of a map projection is then introduced towards an expert system reasoning mechanism. In the reasoning mechanism, map projection selection uses the knowledge base including selection priority conditions and map projection properties to choose the most appropriate map projection (with minimum distortion errors in the view). Figure 6, examples of selecting the appropriate map projection in MMP.



scale: 1:215,026,620	scale: 1:66,144,206	scale: 1:2,263,886
φ = 49.97N	φ = 41.08N	φ = 38.81N
λ = 19.13E	λ = 45.84E	λ =45.34E
shape ≡ straight parallels and meridians	shape≡ straight meridian	shape≡ No straight parallel and meridian
symmetry: double	symmetry: single	symmetry: double
⇓	⇓	⇓
Mercator	LCC(φ = 41)	UTM(zone37)

Figure 6, Samples of selecting the appropriate map projection in MMP

Implemented MMP system considers all mentioned properties for 974 map projections. As it is clear now map projection selection in multi map projection has special properties; these separate it from other expert selection systems.

- Generally, a multi map projection system recognizes view window conditions automatically which before in developed expert systems were map conditions (E.g. position and extent).
- Other conditions, which could not be intelligently recognized like application and geometric properties, are eligible for user modification. On the condition of choosing nothing, system automatically prioritizes these parameters.
- Simultaneously system select suitable map projection as user modifies in the view

6. Discussion AND SUGGESTION

As access to information overlaid over maps become more prevalent and available to the general public through the web and other media, it is increasingly important to have an intelligent map projection for users.

Data exploration may vary from screen map interaction by non-professional users to extensive database manipulation by professional geoscientists. This MMP method is particularly useful when exploring large geographical extents.
In MacEachren's (1994) cartography-cubed (figure 7), where stages were placed upon the communication-visualization diagonal. The MMP select map projection for users so the visualization goes to low interaction with human. Also automatic selection in MMP increases the viewers to a large public group which there is no need to have information about map projections and selection schemas.

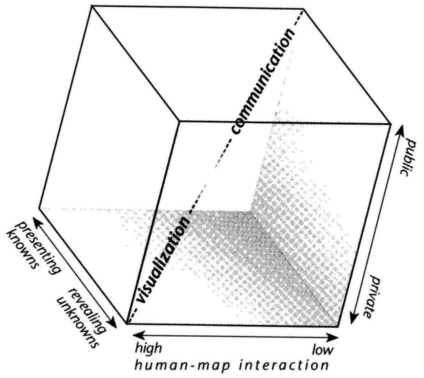


Figure 7, MacEachren's cartography-cubed

Use of different map projections and geodetic referencing systems is one of the important challenges in geospatial data integration and distribution. Using MMP enable us to visualize the geospatial data in created map projection.

6.2. Conclusion

In the presented study, the possibility of continuous display of a limitless global database has been evaluated by simultaneous development of MMP concept and software. Also MMP concept was examined for implementation and geodesic parameters challenges were discussed.

- The parameters considered more in this project were geodesic intervals of map projections; it seems determining these intervals (scale range, geodesic location) are of prime priority in the fully intelligent selection process.
- Scale parameter is among the parameters which have been evaluated with more precision in this project. Due to effecting geodesic location on distortion error determining appropriate map projection on each scale with comparing method at run time would be accurate.
- Due to the fact that datums defined in a specified location, on determining map projections geodesic location, it should be regarded.

There are still challenges in MMP which need future works on following discussions.

- Datum wide-laid-out map and map projection laid-out-map if prepared would have a considerable effect on the accuracy of map projection selection. This map will show appropriate datum boundary per each geodesic location.
- MMP need a specified knowledge base considering its characteristics with wide range of map projections and effecting parameters. Also different artificial intelligent methods must be evaluated to use in MMP selection process.

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